Lecture 14 - Oct. 29

Bridge Controller

Inference Rules: Proof Steps
Interpreting Unprovable Sequents

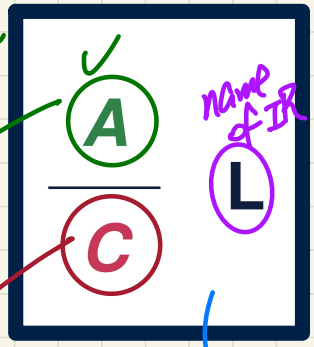
Announcements/Reminders

- ProgTest1 & WT1 results released
- Lab4 released (ProgTest2 on November 6)
  + Scheduled lab session on **October 30**.

Inference Rule: Syntax and Semantics

$\vdash G \equiv True \vdash G$

Syntax



antecedent (a set of sequents)

consequent (a single sequent)

name of IR

Semantics

$A \Rightarrow C \equiv True.$

Q. What does it mean when A is empty/absent?

$\frac{}{C}\ L$ means $\frac{True}{C}$ (meaning that C is an axiom)

To prove the consequent C, it's sufficient to prove the antecedent A (instead). rewriting.

Examples

$$\frac{H_1 \vdash G}{H_1, H_2 \vdash G}\ Mon$$

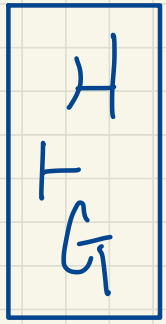
Mon = monotonicity.

you can drop hypotheses

To prove $H_1, H_2 \vdash G$

it's sufficient to prove $H_1 \vdash G$

Sequent



$H \Rightarrow G$

Inference Rule

$$\frac{A}{C}\,L \qquad A \Rightarrow C \equiv True$$

to make progress in proofs, rewrite C into A

Proof of Sequent: Steps and Structure

Outstanding Sequent to Prove

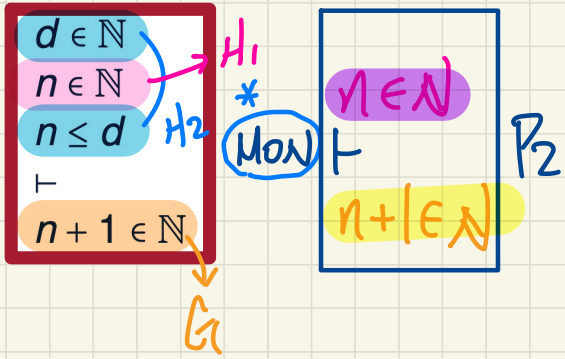
$d \in \mathbb{N}$
$n \in \mathbb{N}$
$n \leq d$
$\vdash$
$n + 1 \in \mathbb{N}$

ML_out/inv0_1/INV

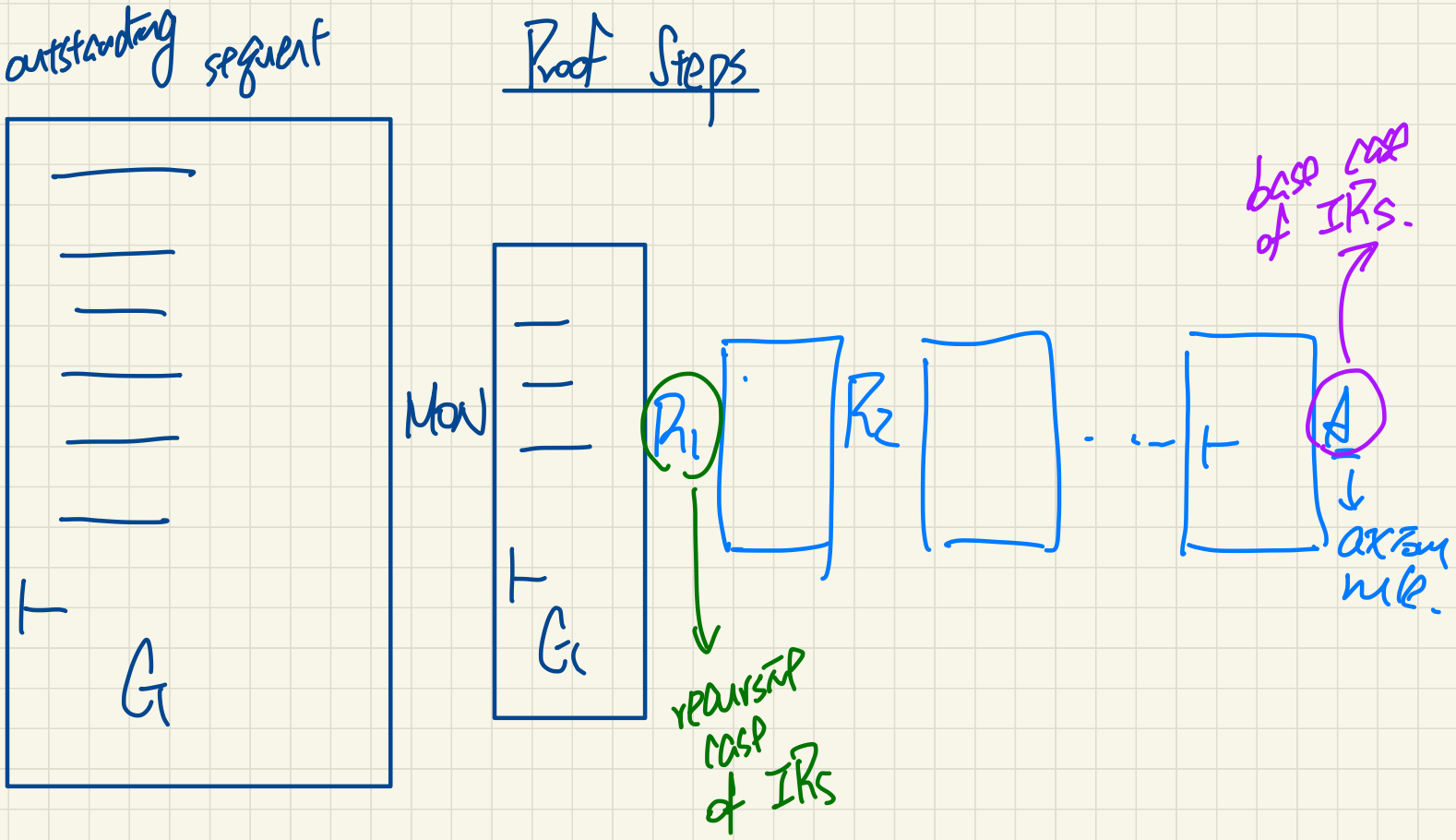
Known Inference Rules

$$\frac{H1 \vdash G}{H1, H2 \vdash G}\ \text{MON}$$

$$\frac{}{n \in \mathbb{N} \vdash n + 1 \in \mathbb{N}}\ \text{P2}$$



* Analyze the goal predicate to prove: what are the relevant variables? what hypotheses are useful?


outstanding sequent
Proof Steps
⊢
G
MON
⊢
G'
R1
R2
⊢
A
base case of IRs.
axiom rule.
recursive case of IRs

Understanding Inference Rule: OR_L

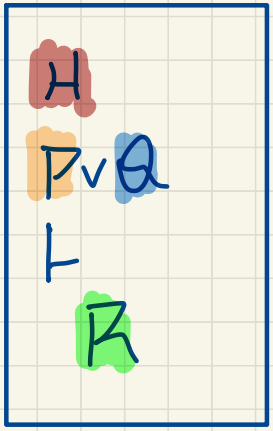
$$\frac{H, P \vdash R \qquad H, Q \vdash R}{H, P \vee Q \vdash R}\ \text{OR\_L}$$

disjunction

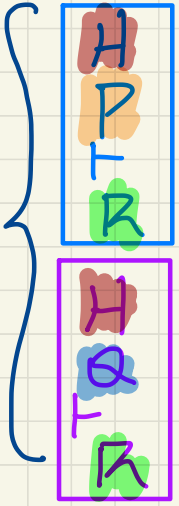
Q1. Does OR_L help us:

(A) split one sequent into two

(B) combine two sequents into one



OR_L



the rule concerns what's to the L of ⊢

Example Inference Rules

Peano Numbers Theorems

Axioms rules (base cases)

$$\frac{}{\vdash 0 \in \mathbb{N}}\ \text{P1}$$

$$\frac{}{n \in \mathbb{N} \vdash n+1 \in \mathbb{N}}\ \text{P2}$$

$$\frac{}{n < m \vdash n+1 \leq m}\ \text{INC}$$

$$\frac{}{0 < n \vdash n-1 \in \mathbb{N}}\ \text{P2'}$$

$$\frac{}{n \leq m \vdash n-1 < m}\ \text{DEC}$$

$$\frac{}{n \in \mathbb{N} \vdash 0 \leq n}\ \text{P3}$$

$n < m$

$\vee$

$n = m$

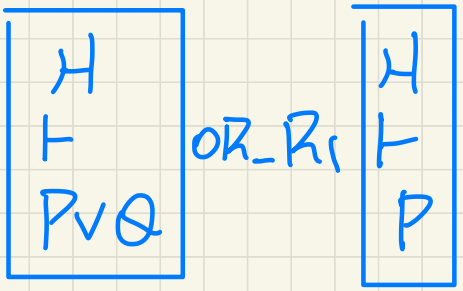
$$\frac{H, P \vdash R \qquad H, Q \vdash R}{H, P \vee Q \vdash R}\ \text{OR\_L}$$

$$\frac{H \vdash P}{H \vdash P \vee Q}\ \text{OR\_R1}$$

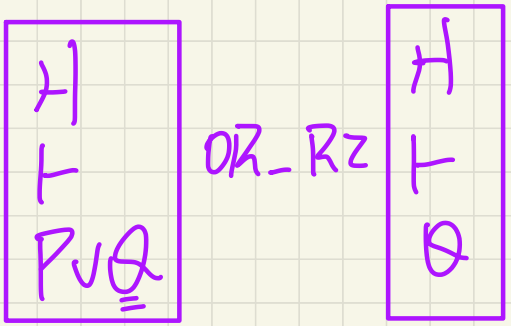
$$\frac{H \vdash Q}{H \vdash P \vee Q}\ \text{OR\_R2}$$

$$\frac{H1 \vdash G}{H1, H2 \vdash G}\ \text{MON}$$

$$\frac{H \vdash P}{H \vdash P \vee Q}\ \text{OR\_R1}$$



$$\frac{H \vdash Q}{H \vdash P \vee Q}\ \text{OR\_R2}$$



Discharging POs of original m0: Invariant Preservation

ML_out/inv0_1/INV

$d \in \mathbb{N}$
$n \in \mathbb{N}$
$n \leq d$
$\vdash$
$n+1 \in \mathbb{N}$

Exercise!

ML_in/inv0_1/INV

$d \in \mathbb{N}$
$n \in \mathbb{N}$
$n \leq d$
$\vdash$ True guard of ML_m1
$n-1 \in \mathbb{N}$

Mon

$n \in \mathbb{N}$
$\vdash$?
$n-1 \in \mathbb{N}$

new guard for ML_m1: $n > 0$

unprovable

One possible resolution: add some guard condition of ML_m1, so that the extra hypothesis can make this seq. provable.

$$\frac{H \vdash P}{H \vdash P \vee Q}\ \text{OR\_R1}$$

$$\frac{H1 \vdash G}{H1, H2 \vdash G}\ \text{MON}$$

$$\frac{}{n \leq m \vdash n-1 < m}\ \text{DEC}$$

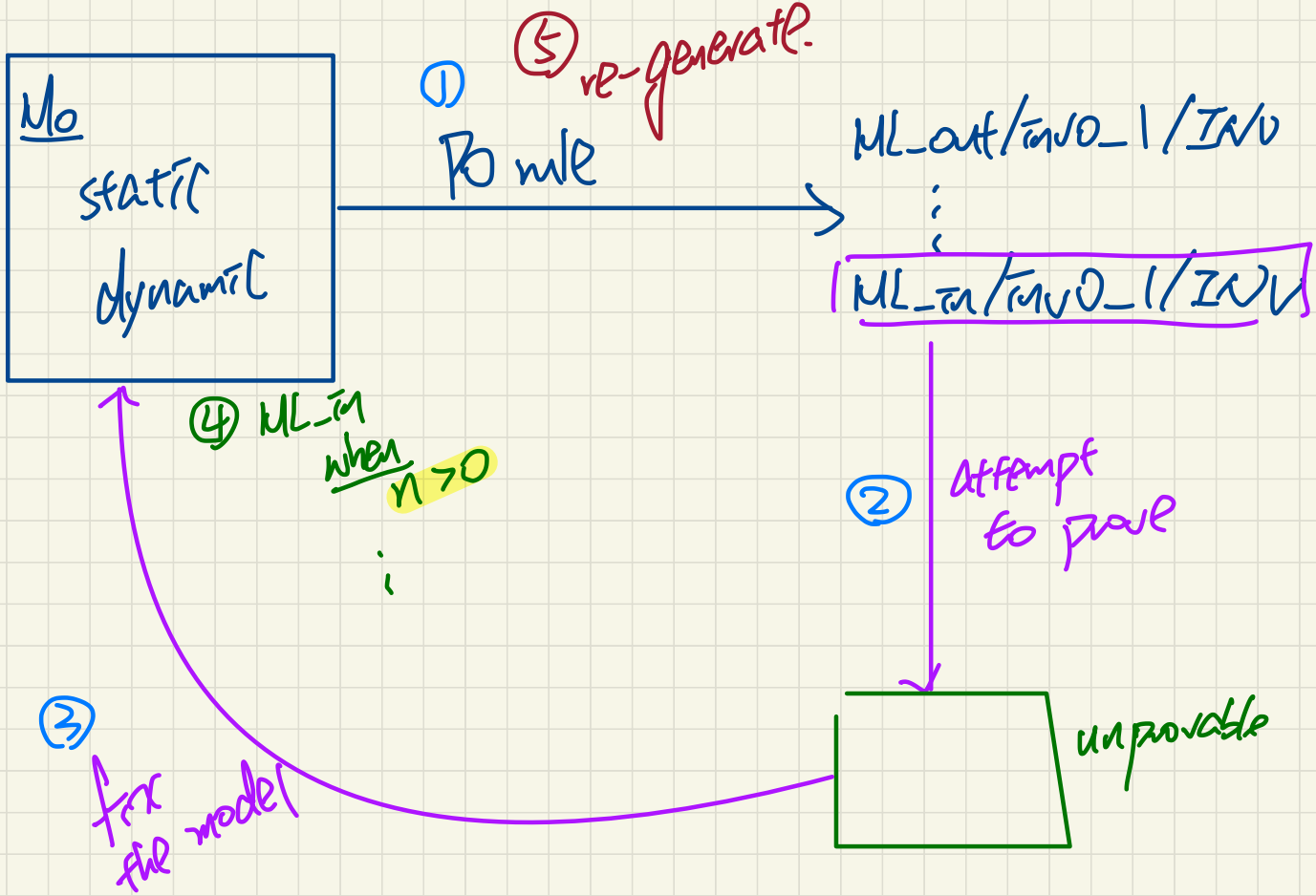
$$\frac{}{n \in \mathbb{N} \vdash n+1 \in \mathbb{N}}\ \text{P2}$$

ML_out/inv0_2/INV

$d \in \mathbb{N}$
$n \in \mathbb{N}$
$n \leq d$
$\vdash$
$n+1 \leq d$

ML_in/inv0_2/INV

$d \in \mathbb{N}$
$n \in \mathbb{N}$
$n \leq d$
$\vdash$
$n-1 \leq d$


M0
static
dynamic
① Rule
⑤ re-generate
ML_0/inv0_1/INV
ML_n/inv0_1/INV
② attempt to prove
unprovable
③ for the model
④ ML_n when n>0